Factorisation en matrices (semi-)non-négatives : problématique, applications et tendances

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Travail collaboratif incluant au LISIC

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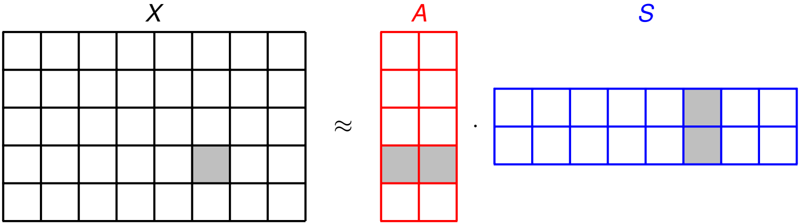
Foreword: matrix factorization

Many problems in *machine learning* and in *signal/image processing* can be rewritten as a system of equations $X \approx A \cdot S$:



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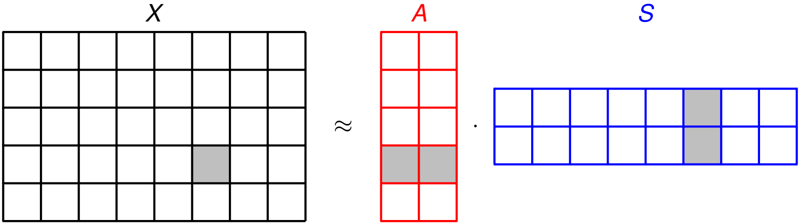

source localization/separation

X observed data matrix, A mixing matrix, S source matrix

- If A estimated (and sensor array geometry is known)
- ➪ mixture estimation (source localization)
- If S estimated (and sensor array geometry is known)
- ➪ source separation (beamforming)

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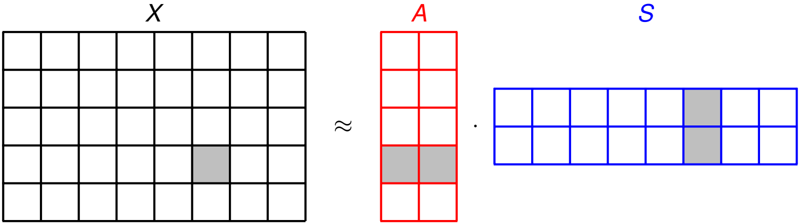

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- Is source separation a Signal Processing or an AI topic? Ask Google (Google AI blog submitted to SIGGRAPH'18).

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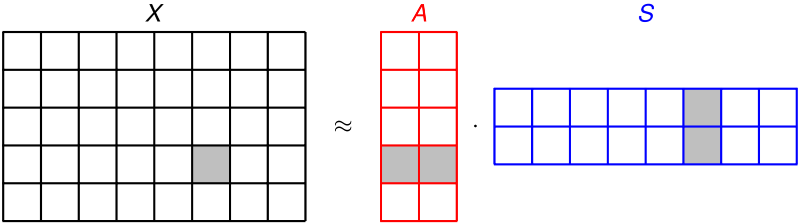


Low-rank matrix approximation

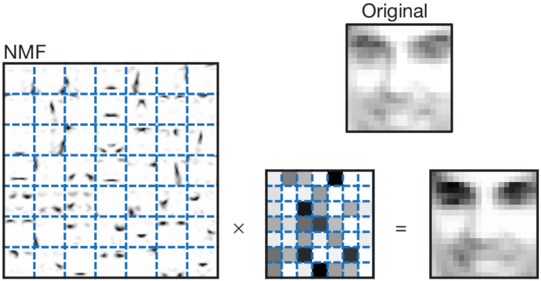
X noisy matrix (possibly with missing entries), A weight matrix, S latent variable matrix

- Topic modeling (e.g., Google News)
- Collaborative filtering (e.g., Netflix Prize)
- Graph analysis (after a transformation into a matrix – e.g., yellow cab / velib network)
- Mobile sensor calibration (Ph.D. theses of Clément Dorffer & Farouk Yahaya)

NMF: why is it so popular?

- In many problems, matrices $X \approx A \cdot S$ are non-negative
  - chemical source separation
  - hyperspectral imagery
  - mobile sensor calibration

➩ Non-negativity on A and/or S yield **better interpretability**



NMF applied to face dataset (source: Lee & Seung, 1999)

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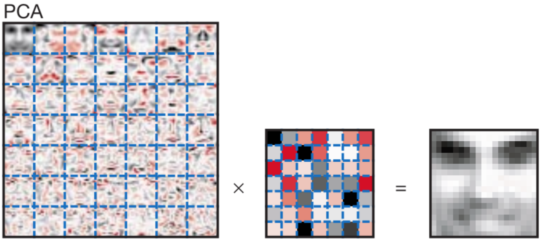
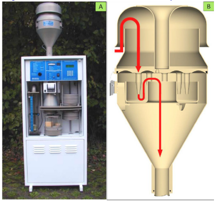


Principal Component Analysis applied to face dataset (source: Lee & Seung, 1999)

Application (1)

Separation of pollutant sources (Limem *et al.*, 2012–2014, Chreiky *et al.*, 2015–2017)



- Collaborative work with UCEIV (D. Courcot, F. Ledoux)
- Observation of air-suspended fine particulate matter over time
- Particles are analyzed by chemists who provide chemical concentrations of each species (e.g., iron, zinc, calcium, sodium, etc.) in ng/m^3
- ➪ Information stored in $X \simeq A \cdot S$
- Observed data are mixtures of "chemical profiles" (=chemical signatures which are specific to each emission source – factory chimney, exhaust, marine particles, etc – lines of S)
- We proposed **Informed** NMF : using partial expert's (in S) and meteorological knowledge (informing A) to improve the separation enhancement
- Application on Dunkirk air quality

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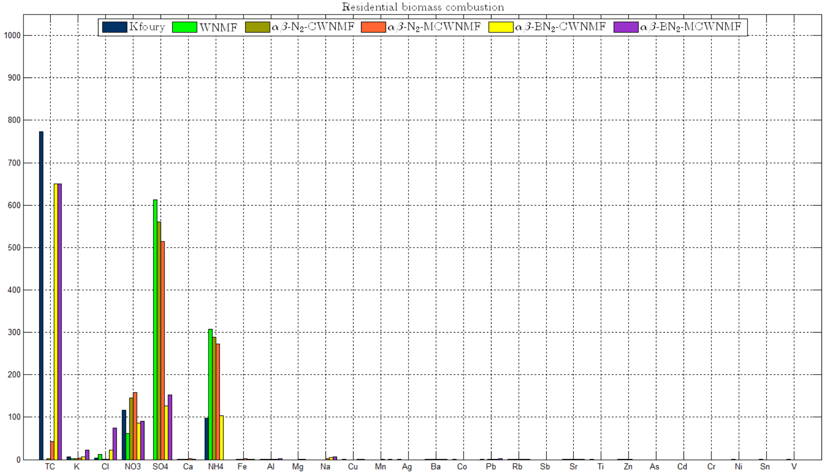

On estimated source profile (from S)

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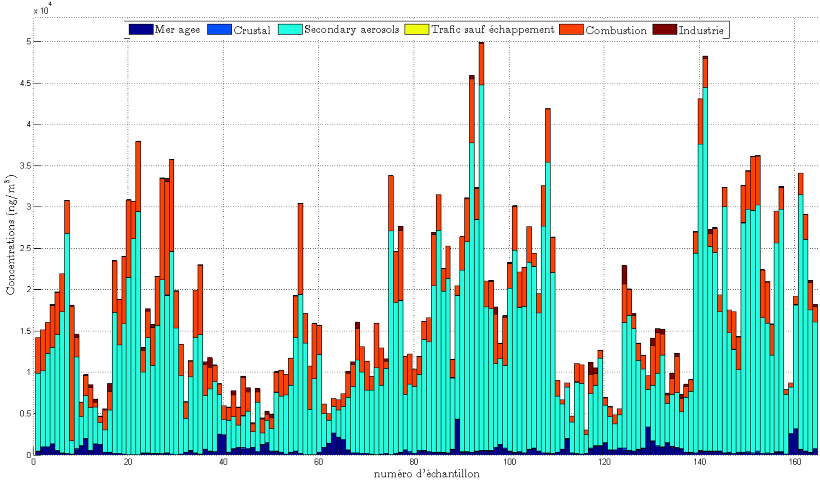

Global origin of sensed PM with respect to time (*A*)

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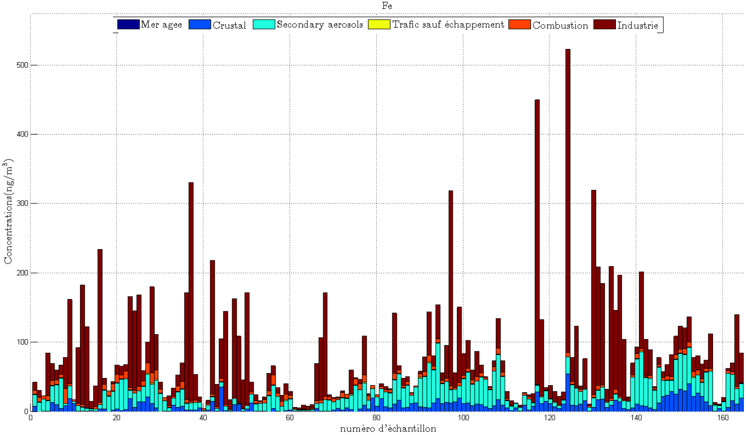

Contribution of iron source in PM (from *A*)

Application (2)

Mobile sensor calibration (Dorffer *et al.*, 2015–2018)

- Sensed phenomenon $\Longrightarrow$ voltage
- Voltage $\Longrightarrow$ phenomenon?

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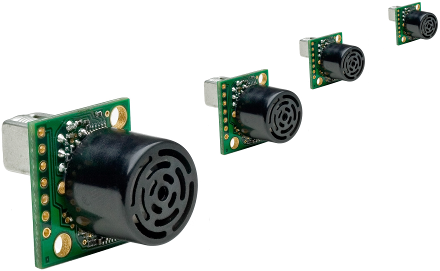
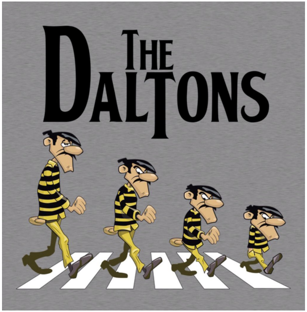
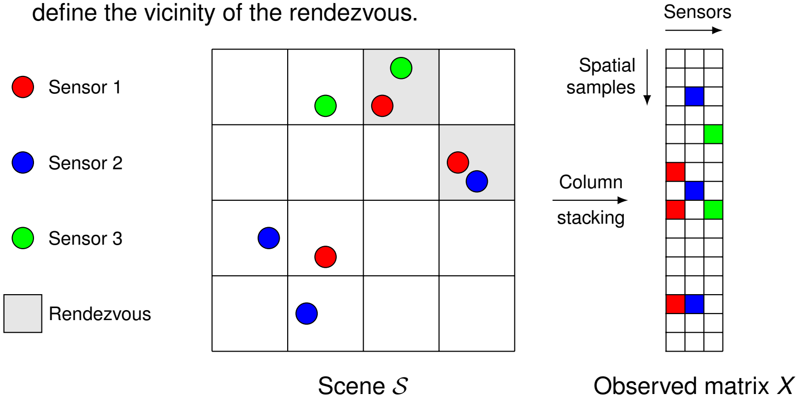




- Sensed phenomenon $\Longrightarrow$ voltage
- Voltage $\Longrightarrow$ phenomenon?
  - Sensor calibration needed
  - Not always physically possible
  - ➯ **Blind sensor calibration**

Application (2)
Definitions

- A rendezvous is a temporal and spatial vicinity between two sensors (Saukh *et al.*, 2013).
- A scene $\mathcal{S}$ is a discretized area observed during a time interval $[t, t+\Delta t)$. A spatial pixel has a size lower than Δd, where Δt and Δd define the vicinity of the rendezvous.



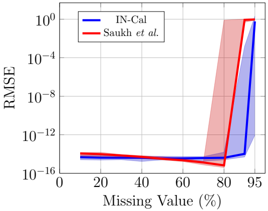
Blind calibration revisited as a weighted NMF problem (affine model)

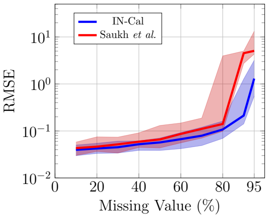
$$W \circ \underbrace{\begin{bmatrix} x(1,1) & \cdots & x(1,m-1) & y(1) \\ \vdots & & \vdots & \vdots \\ x(n,1) & \cdots & x(n,m-1) & y(n) \end{bmatrix}}_{X} \simeq W \circ \left(\underbrace{\begin{bmatrix} y(1) & 1 \\ \vdots & \vdots \\ y(n) & 1 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_{m-1} & 1 \\ \beta_1 & \beta_2 & \cdots & \beta_{m-1} & 0 \end{bmatrix}}_{S} \right)$$

We proposed methods:

- adding information on A (sparse assumptions) and S (sensor information)
- handling more complex calibration models (e.g., nonlinear)

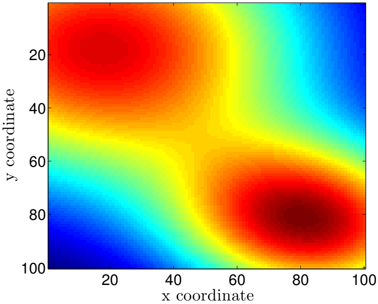
Estimation of S



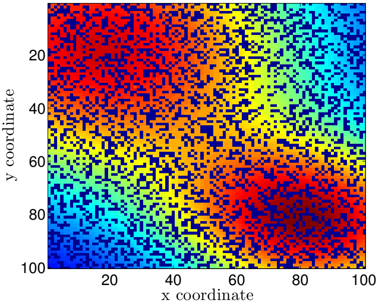


Estimation of *A*

Theoretical scene



First informed NMF (IN-Cal)

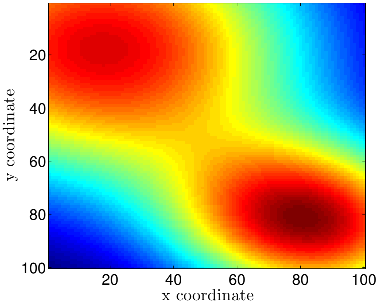


Application (2)

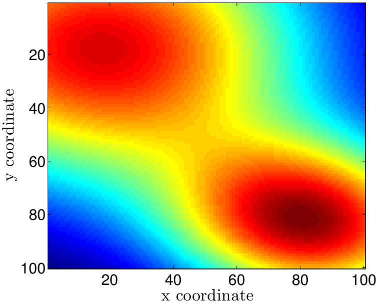
Reconstruction accuracy

Estimation of *A*

Theoretical scene



+ using sparse assumptions on *A* (SpIN-Cal)



The future of NMF... in LISIC

We now briefly discuss some of our theoretical work directions for the next 5 years:

1. being fast to process the data deluge (Ph.D. thesis of F. Yahaya)

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Being fast to process the data deluge

Historical NMF techniques and their extensions (including some of ours)

- Using slow techniques for the update rules (MU)

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Fastening NMF

- Distributed computing (e.g., Liu *et al.*, 2010)
- Online factorization (e.g., Mairal *et al.*, 2010)
- Fast solver (e.g., Guan *et al.*, 2012, Dorffer *et al.*, 2017)
- Randomized strategies (e.g., Zhou *et al.*, 2012 or Tepper & Sapiro, 2016,

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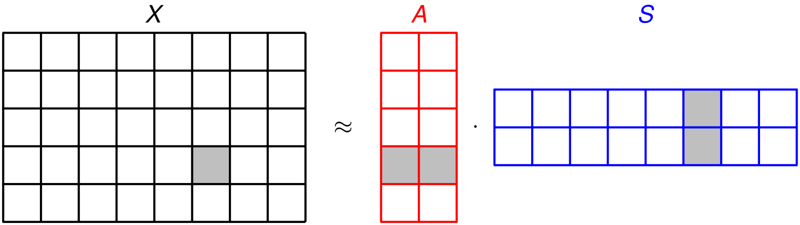
Randomized strategies for NMF

Key idea

- Compress some matrices in the NMF problem in order to reduce its size



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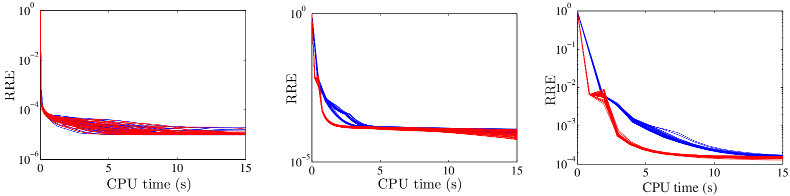
Reconstruction of rank-15 matrices of size 500×500 (left), 5000×5000 (middle) and 10000×10000 (right). Perf criterion:

$$\text{RRE} = \frac{\|X - AS\|_{\mathcal{F}}}{\|X\|_{\mathcal{F}}}$$



- In blue: the fast NeNMF method
- In red: our proposed faster-than-fast randomized NeNMF

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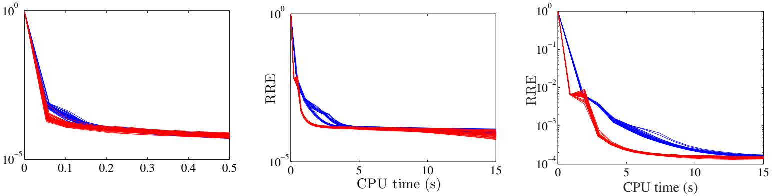

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Conclusion

- NMF $=$ powerful tool for signal, image processing and machine learning
- Good expertise in LISIC, with methodological developments
- Still many ideas and room for innovative work
- **Questions?**