

# Surrogate models for simulation based problems

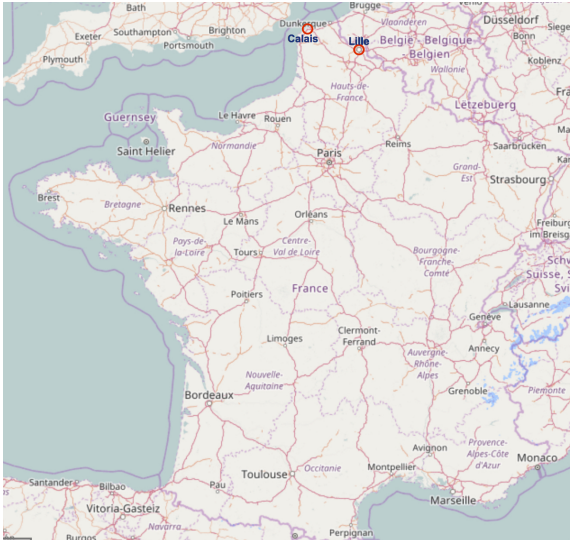
SÉBASTIEN VEREL

Laboratoire d'Informatique, Signal et Image de la Côte d'opale (LISIC)  
Université du Littoral Côte d'Opale, Calais, France  
<http://www-lisic.univ-littoral.fr/~verel/>

Master on Software Engineering and Artificial Intelligence,  
University of Malaga,  
November, 29, 2022



# Where am I from ?



# Situation map : Université du Littoral Côte d'Opale

*LISIC : laboratory of computer science, signal, image*

Boulogne (fishing port), Calais (transportation port),  
Dunkerque (industrial port), Saint Omer (glass industry).



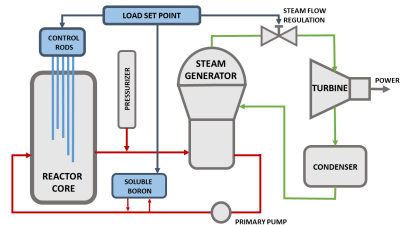
↔ Lille (0h30), London (1h), Bruxelles (1h15), Paris (1h30), Amsterdam (2h)

# Context

## Mobility system



## Nuclear energy system



A priori, each domain is very different

But, share :

- Design problems for new perspectives,
- Inaccessible (cost) quantities, scales, etc.



# Solving design problems, etc.

## Solving optimization problems (mono- or multi-objective)

- Using the cognitive, and social abilities of humans :  
expert knowledges, evaluation of risk, uncertainties, divide into sub-problems, complex reasoning, etc.
- Using the computational, and memorization abilities of machines :  
automatic, data, formal language, speed, multi-scale, etc.

## Main AI approaches for automatic solving

- Algebraic approach : algebraic, or formel model
- Digital twin approach :  
numerical model, and numerical simulation

# Comparison of approaches

## Algebraic approach

- Formal model
- Aggregated variables, noise (demand, uncertainties), constraints,...
- Artificial or real-like problem instances
- Offline

Tools :

cplex, gurobi, constr. prog.,  
local search, ea, etc.

$$\max \sum_{i \in T} (p_i f_i) - \sum_{i \in T} \sum_{o \in O} (s_{i,o} u_{i,o} + S s_{i,o})$$

$$s_{i-1,o} + b_{i,o} = u_{i,o} + s_{i,o} \quad \forall i \in T \setminus I_0, \forall o \in O$$

$$i + b_{i,o} = u_{i,o} + s_{i,o} \quad \forall o \in O$$

$$s_{i,o} = i \quad \forall o \in O$$

$$\sum_{o \in V} u_{i,o} \leq 200 \quad \forall i \in T$$

$$\sum_{o \in V} u_{i,o} \leq 250 \quad \forall i \in T$$

$$3f_i \leq \sum_{o \in V} h_{o,i} u_{i,o} \leq 6f_i \quad \forall i \in T$$

$$\sum_{o \in V} u_{i,o} = f_i \quad \forall i \in T$$

$$b_{i,o}, u_{i,o}, s_{i,o} \geq 0 \quad \forall i \in T, \forall o \in O$$

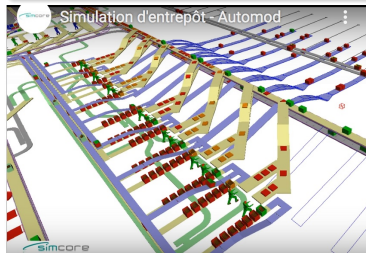
$$f_i \geq 0 \quad \forall i \in T$$

## Digital twin approach

- Low level model
- Complex interactions
- Flow of data : sensor, etc.
- Offline, Online

Tools :

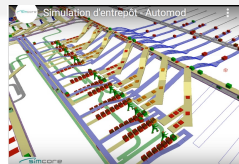
simcore, simio, matsim, devs,...



# Consequences for automatic solving

$$\begin{aligned} & \max \sum_{i \in T} (pf_i) - \sum_{i \in T} \sum_{o \in O} (g_{i,o} u_{i,o} + 5s_{i,o}) \\ s_{i-1,o} + b_{i,o} &= u_{i,o} + s_{i,o} \quad \forall i \in T \setminus t_0, \forall o \in O \\ i + b_{i,o} &= u_{i,o} + s_{i,o} \quad \forall o \in O \\ s_{i,o} &= i \quad \forall o \in O \\ \sum_{o \in V} u_{i,o} &\leq 200 \quad \forall i \in T \end{aligned}$$

$$\begin{aligned} \sum_{o \in V} u_{i,o} &\leq 250 \quad \forall i \in T \\ 3f_i &\leq \sum_{o \in V} h_{i,o} u_{i,o} \leq 6f_i \quad \forall i \in T \\ \sum_{o \in V} u_{i,o} &= f_i \quad \forall i \in T \\ b_{i,o}, u_{i,o}, s_{i,o} &\geq 0 \quad \forall i \in T, \forall o \in O \\ f_i &\geq 0 \quad \forall i \in T \end{aligned}$$



## Algebraic approach

Pros :

- Exploitation of the algebraic properties (fast to compute)
- Explicit, and synthetic model

Difficulties :

- Design of the model :  
creation of languages, etc.

## Digital twin approach

Pros :

- Low level description
- Tests, visualization

Difficulties :

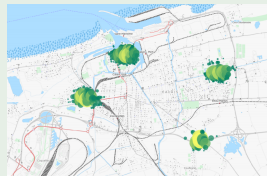
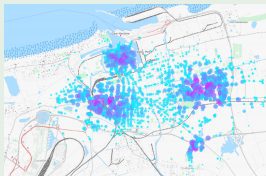
- $\approx$  Black-box :  $(x, f(x))$
- Costly simulation  
(time, energy)

Indeed, not only "solving", but also support of decision making :  
before, during, and after the optimization process

# Digital twin for mobility system

F. Leprêtre, V. Marion, C. Fonlupt, S. Verel (LISIC) - thesis 2017 - 2020.  
H. Aguirre, R. Armas, K. Tanaka (Shinshu Univ., Nagano, jp)  
Partner : Calais City, Marie Capon, (expertise, and funding)

## SIALAC benchmark of mobility



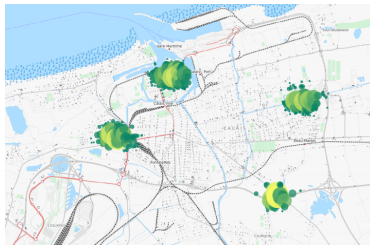
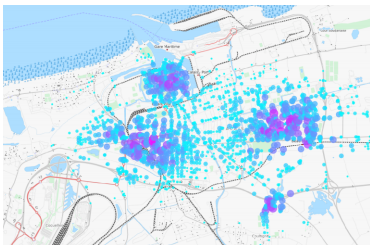
Different and, futur scenario : home, agents, activity

## Two problems

- Tuning of traffic light
- Bus stop position

# SIALAC benchmark of mobility

Leprêtre, F., et al. Applied Soft Computing, 2019 [12]



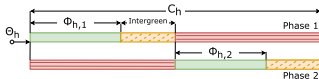
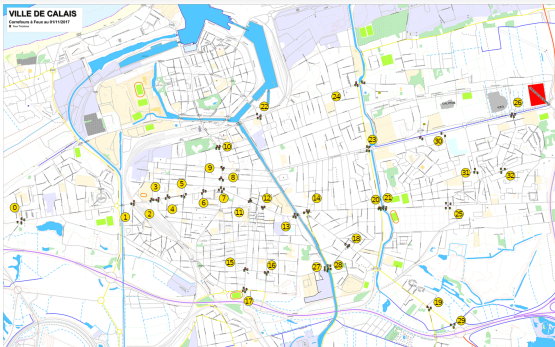
Number of agents	$\{5, 10, 15, 20\} \times 10^3$
Home	{1 cluster, 4 clusters, uniform}
Activity	{1 cluster, 4 clusters}
Signals systems	{50%, 75%, 100%}

72 scenario using MatSim (Multi-Agent Transport Simulation)

## Goal

- Show to the partner what it is possible with such tools
- Design **robust** optimization algorithms for mobility problems

# Traffic light problem for Calais, and Quito cities



- Space : 33 (Calais, France), 70 (Quito, Equator) intersections  
search space dim.  $\times 4$  integer variables
- Criteria : minimize average travel time (black-box problem)
- Computational time per simulation  $\approx 1$  minute

# Gradient free optimization algorithms

## Stochastic Hill Climber

```
 $x \leftarrow$  initialize random solution  
repeat  
  
   $x' \leftarrow$  mutate  $x$   
   $x \leftarrow x'$  if  $f(x') < f(x)$   
until stopping criterion met
```

## Evolutionary Algorithm

```
 $P = \{x_1, \dots, x_\mu\} \leftarrow$  rnd. init.  
repeat  
   $P_{genitor} \leftarrow$  selection from  $P$   
   $P_{children} \leftarrow$  breed  $P_{genitor}$   
   $P \leftarrow$  replace  $P \cup P_{children}$   
until stopping criterion met
```

- mutate : random variation of candidate solution
- Tradeoff exploration / exploitation : mutate / selection

How to tune the mutation operator ?  
*i.e.* Where to explore ?

## Surrogate, and model-based approaches

According to the context, the search strategy can be different.

When the evaluation time of a single candidate solution is :

- short : try, and test strategy (local search, EA, etc.)  
a test is fast, so multiple tests are possible.  
Memory "less" strategy.  
ex. : re-computation of a solution
- long : model based strategy  
spend more time to design a new candidate solution,  
aggregation of information on the problem (model), and test  
ex. : 200 evaluations available on problem of dimension 100



# Structure of real-world problems

## Intuitively

Real-world problem instances are often "structured" :

- Local sub-problems are not random,
- Interdependency between sub-problems are not random.

## Importance of variables

Consequence : some variables are more impactful than others.

## Examples

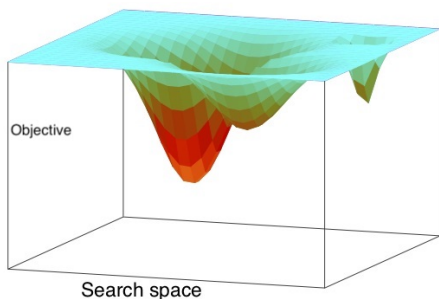
Isolated traffic lights are less impact on travel time than central traffic lights

How to detect important variable to design a model of problem ?  
Expert knowledge, or more automatically....

# Fitness landscape : a model of the search space

## Fitness landscape (Wright 1920)

- $\mathcal{S}$  : set of candidate solutions, search space
- $f : \mathcal{S} \rightarrow \mathbb{R}$  : objective function
- $\mathcal{N} : \mathcal{S} \rightarrow 2^{\mathcal{S}}$ , neighborhood relation between solutions



- Geometry of the fitness landscape :  
Features/metrics  
are correlated to  
algorithm performance

⇒ Toward automatic design (tuning/control) of algorithms

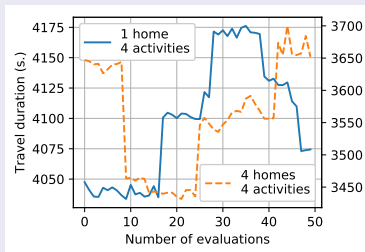
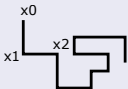
# Offline model of problem

Importance degree of variable  $i$

$$\delta_i = |f(\text{mutate}_i(x)) - f(x)|$$

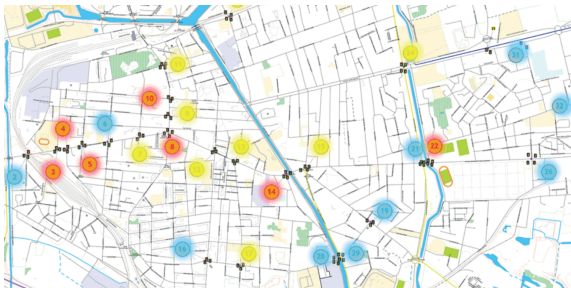
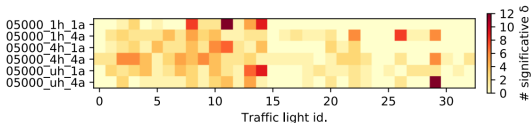
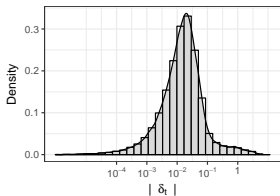
Estimation : Random walk on fitness landscape

Sequence of neighboring solutions :  $(x_0, x_1, x_2, \dots)$



Offline model, before running the algorithm (expert knowledge?) :  
Importance  $\delta_i$  associated to each var.  $i$

# Add explainable model to optimization



⇒ Explainable model (cf. XAI) defined with "score",  
Improve the communication with the partners

# Adaptive algorithm based on offline model

Backbone in combinatorial problems :

"good" solutions have some specific variables value

## Design of mutation operator

- **Hypothesis** : modify in priority important variables
- **Goal** : automatic learning of expert knowledge

## Method

- Divide the set of variables into 3 groups according to importance
- Use reinforcement machine learning technique to select the group to mutate.

# Adaptive bandit descent

## Multi-armed bandit problem (reinforcement learning)

UCB strategy to select to relevant arm :

$$\hat{r}_i + C \sqrt{\frac{S}{s_i}}$$

$\hat{r}_i$  : reward,  $s_i$  : nb. of selection of arm  $i$ , and  $S$  : total nb. of selection,  $C$  : tradeoff parameter



## Adaptive algorithm

$G \leftarrow$  split var. into groups

$x \leftarrow$  initialize random solution

**repeat**

$g' \leftarrow$  select group in  $G$  using UCB rule

$x' \leftarrow$  mutate a variable from  $g$  of  $x$

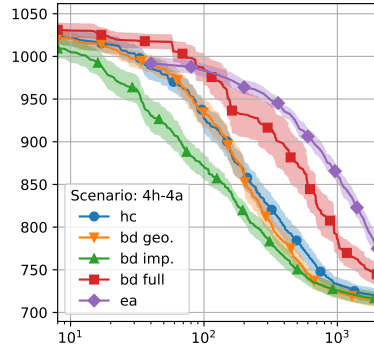
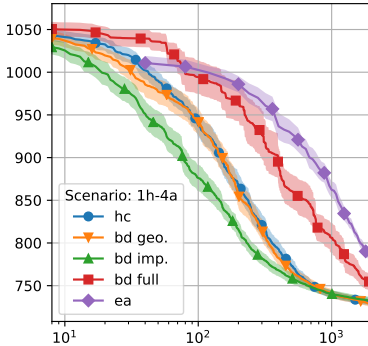
$x \leftarrow x'$  **if**  $f(x') < f(x)$

    Update rewards

**until** stopping criterion met

# Some results

Quality vs. number of evaluations :



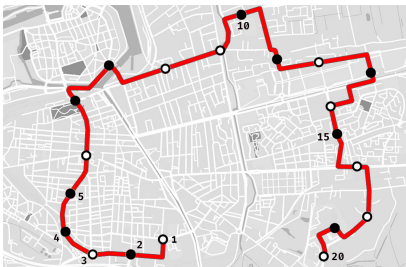
- Speed up the convergence
- Better than "hand made" groups, or previous Evolutionary Algorithm
- Robust on different scenario (also for Quito city)

# Optimisation with time expensive simulation

- Parallel computation : distribute computation on machines
- **Surrogate model** : online substitution of the original function with an (approximated) function fast to compute

## Surrogate model

- A lot of works on numerical optimization :  
 $f : \mathbb{R}^d \rightarrow \mathbb{R}$
- Few works on discrete/combinatorial optimization :  
 $f : \{0, 1\}^d \rightarrow \mathbb{R}$ , or  $f : \mathcal{S}_n \rightarrow \mathbb{R}$



## Bus stop position problem

- Space :  $\{0, 1\}^d$  open or close possible stops
- Criterium : min. travel time



# Surrogate-assisted opt. of pseudo-boolean problems

Florain Leprêtre, Virginie Marion, Cyril Fonlupt (LISIC),

K. Tanaka, H. Aguirre (Univ. Shinshu), A. Liefooghe, B. Derbel (univ. Lille)

## Surrogate-Assisted Optimization

$X \leftarrow$  initial sample

**repeat**

$M \leftarrow$  Build model of  $f$  from  $X$

$x^* \leftarrow$  Optimize *w.r.t.* an acquisition function based on  $M$

$y^* \leftarrow f(x^*)$  using the numerical simulation

$X \leftarrow X \cup \{(x^*, y^*)\}$

**until** time limit

In numerical optimization [18] :

- Models :  
Gaussian Process, polynomial chaos, NN, RBF, RF, deep\*, etc.
- Acquisition function :  
 $M$ , Expected improvement, probability impr., UCB, etc.

In discrete optimization [2] :

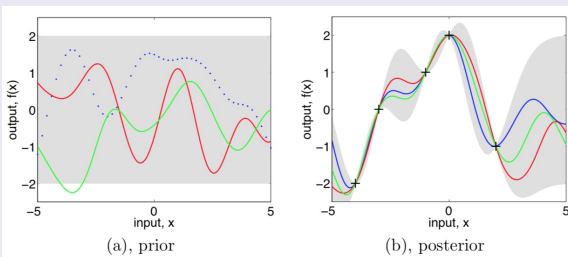
- Use discrete distance, or numerical variable

# Example : Efficient Global Optimizer [9] [20]

- Model : Gaussian Process  $M(x) \approx \mathcal{N}(m(x), K(x, x'))$
- Acquisition function : Expected Improvement

GP : Random variables which have joint Gaussian distribution.

mean :  $m(y(x)) = \mu$  ; covariance :  $cov(y(x), y(x')) = \exp(-\theta \text{dist}(x, x')^p)$



from : Rasmussen, Williams, GP for ML, MIT Press, 2006.

pros : estimation of uncertainty (expected improvement etc.)

cons : estimation is costly, and distance in high distance is not informative

# Polynomial regression model

## Polynomial chaos regression (PRC)

### Model

A basis of functions  $\{\varphi_j : j \in \{1, \dots, p\}\}$

$$M(x) = \sum_{j=1}^p \beta_j \varphi_j(x)$$

Regression using least square method, or bayesian approach

### Example : second-order polynomial

$$M_2(x) = \beta_0 + \sum_{i=1}^d \beta_i x_i + \sum_{i=1}^{d-1} \sum_{j=i+1}^d \beta_{ij} x_i x_j$$

Pros :

Easy interpretation (XAI), fast to compute, polynomial regression

Cons :

Use a relevant basis of functions (Fourier transform, etc.)

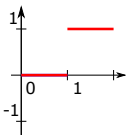
Number of terms increases exponentially with order (sparse methods)

# Pseudo-boolean surrogate : Walsh functions

- Space pseudo-boolean function is a vector space
- Basis : multi-linear functions,  $x_{k_1} \dots x_{k_\ell}$  [Baptista, Poloczek, BOCS, ICML 2018][1]

**Multi-linear :**

$$d = 1, \psi_1(x) = x$$

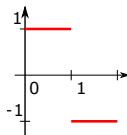


Orthogonal : No

$x$	$\psi_0$	$\psi_1$
0	1	0
1	1	1

**Walsh :**

$$d = 1, \varphi_1(x) = (-1)^x$$



Orthogonal : Yes

$x$	$\varphi_0$	$\varphi_1$
0	1	1
1	1	-1

Extension to dimension  $d$  using tensorial product :

$$\psi_{k_1 \dots k_\ell}(x) = x_{k_1} \dots x_{k_\ell}$$

$$\varphi_{k_1 \dots k_\ell}(x) = (-1)^{x_{k_1}} \dots (-1)^{x_{k_\ell}}$$

# Surrogate model for pseudo-boolean functions

## Walsh functions

$$\forall x \in \{0, 1\}^d, \varphi_k(x) = (-1)^{\sum_{j=0}^{d-1} k_j x_j}$$

## Normal, and orthogonal basis

Any function can be written as :

$$f(x) = \sum_{k=0}^{2^d-1} \beta_k \cdot \varphi_k(x)$$

with :  $\beta_k = \frac{1}{2^d} \sum_{x \in \{0,1\}^d} f(x) \cdot \varphi_k(x)$

Example with order 2, model limited to quadratic interactions :

$$f(x) = \beta_0 + \sum_{i=1}^d \beta_i \cdot \sigma_i + \sum_{i < j=1}^d \beta_{ij} \cdot \sigma_i \sigma_j \text{ with } \sigma_i = (-1)^{x_i}$$

## Why Walsh functions ?

$$f(x) = \beta_0 + \sum_{i=1}^d \beta_i \cdot \sigma_i + \sum_{i < j} \beta_{ij} \cdot \sigma_i \sigma_j \text{ with } \sigma_i = (-1)^{x_i}$$

- Explicit algebraic model (not black-box) : easy to interpret  
Interaction between variables, intensity of interaction  $|w_{i,j}|$
- Efficient algorithms to optimize such problems
- Model of function used in quantum computing  
Also know as Spin-Glasses, or QUBO / UBQP problems [8]

# Surrogate model based on Walsh functions

Expansion to order  $\ell$  (cf. polynomial chaos, sparse grid, etc.)

$$M(x) = \sum_{k : \text{ord}(\varphi_k) \leq \ell} \hat{\beta}_k \cdot \varphi_k(x)$$

- Pros :  
See previous slides
- Cons :  
model dimension (quadratic, cubic, etc.)  
No uncertainty estimation

Estimation of coefficients :

linear regression using sparse techniques : LARS/LASSO, etc.

LASSO :  $\hat{\beta} = \text{argmin}((M(x_i) - y_i)^2 + \alpha \|w\|_1)$

# Walsh Surrogate-assisted Optimizer (WSaO)

## Surrogate-Assisted Optimization

$X \leftarrow$  initial sample

**repeat**

$M \leftarrow$  Build Walsh model of  $f$  from  $X$

$x^* \leftarrow$  Optimize  $M$  using Eff. Hill-Climber

$y^* \leftarrow f(x^*)$  using the numerical simulation

$X \leftarrow X \cup \{(x^*, y^*)\}$

**until** time limit

## Efficient optimization algorithm for Walsh functions

using the additive property :

$$\delta_i(x) = M(x \oplus i) - M(x) = -2 \sum_{k \supset i} \beta_k \varphi_k(x)$$

$$\delta_{ij}(x) = \delta_i(x \oplus j) - \delta_i(x) = 4 \sum_{k \supset i \& k \supset j} \beta_k \varphi_k(x)$$

Find best improving move in  $O(\ell)$  at each step of the search.

Partition crossover to combine 2 solutions

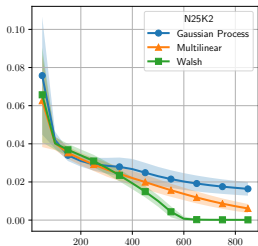
Chicano, Whitley, Ochoa, and Tinós. "Optimizing one million variable NK landscapes by hybridizing deterministic recombination and local search." In Genetic and Evolutionary Computation Conference, 2017. [3]



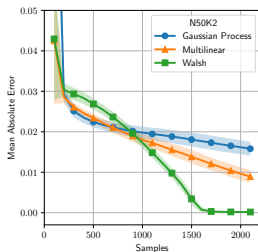
# Quality of Walsh regression on academic benchmarks

Mean abs. error on NK-landscapes benchmark

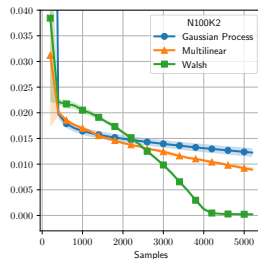
$d = 10$



$d = 25$

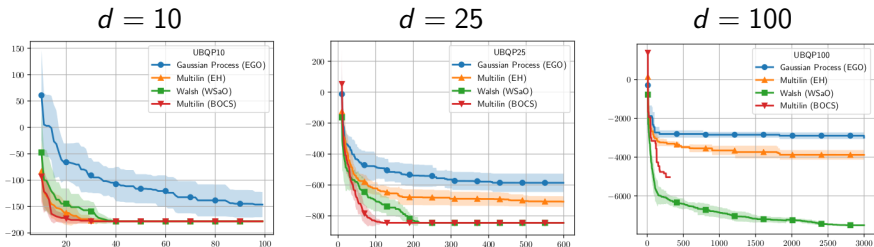


$d = 100$



# Walsh Surrogate-assisted Optimizer (WSaO)

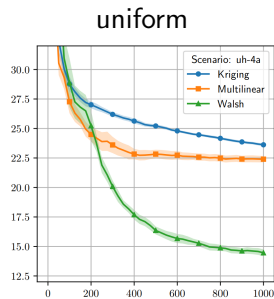
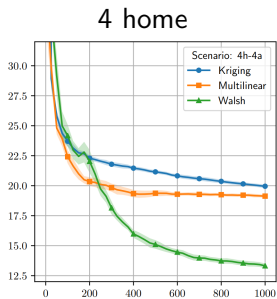
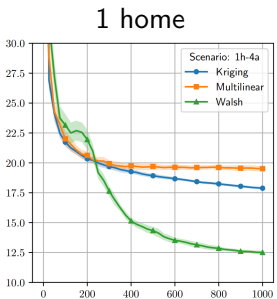
## Performance on UBQP benchmark



- Krigging : information of distance decreases with dimension
- BOCS : bayesian estimation of multilinear basis, SA opt. alg. (very expensive to compute)

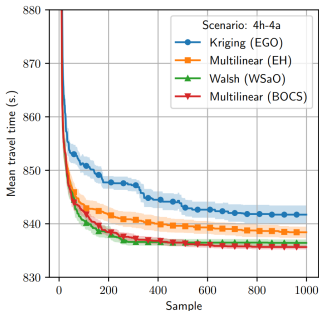
# Preliminary results on bus stop problem

Mean abs. error on instances with  $d = 20$ , 4 activity centers



# WSaO on bus stop problem

Preliminary results for small dimension  $d = 20$  problem



The work is progressing on real data :

Valentin Vendi, PhD student, 2021-2024, "Design of decision-making tools for sustainable mobility in the Hauts-De-France region", co-direction with C. Fonlupt.

Others master student positions, and possible PhD position coming soon, please contact me.

## Comments with surrogate models

- Result with surrogate assisted optimization :
  - Near optimal solution, and an explicit model of your problem
  - Use non black-box machine learning model are useful !
- Open issues :
  - Tradeoff between quality of the model (uncertainty), and optimization effort
- Perspectives :
  - multi-objective optimization, uncertainty, permutation space, numerical & discret, large scale, etc.

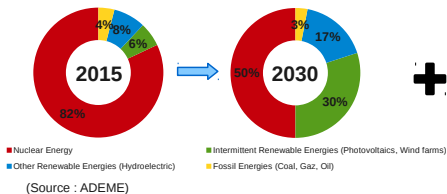
# Context

## Joined work

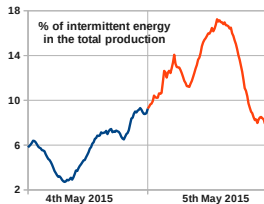
Jean-Michel Do, Jean-Charles Le Pallec, Cheikh Diop, CEA Saclay,  
PhD : Mathieu Muniglia (2014 - 2017), Valentin Drouet (2017 - 2020),  
Baptiste Gasse (2020 - 2023)

Multi-objective optimization of nuclear power plant control for load following in the context of energy transition using evolutionary algorithms

# Context



Large scale deployment of **intermittent renewable energies** in France

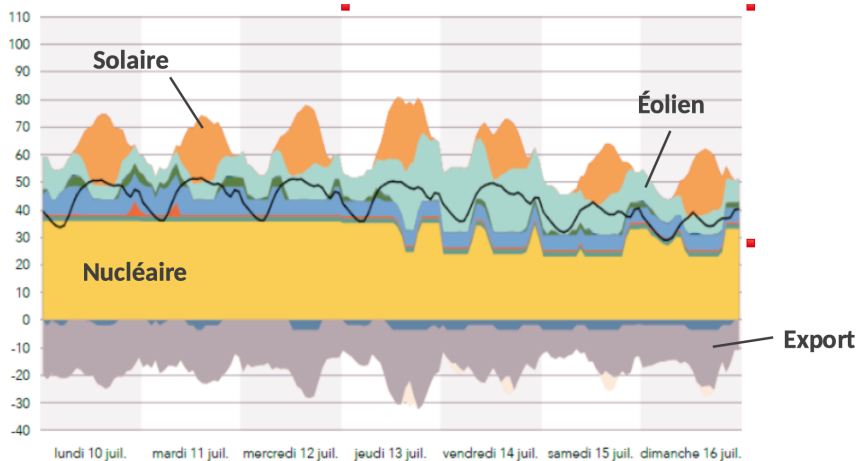


**Highly fluctuating** production rate (up to 3 times the average)

Possible solutions of intermittency :

- Flexibility (on demand)
- Smart grid
- Storage
- Manageability of Pressurized Water Reactors

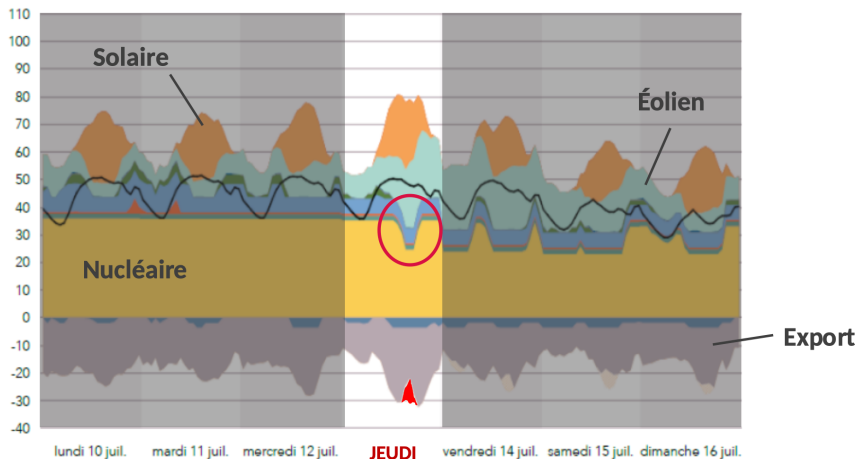
# Scenario of energetic transition in France



RTE (french electricity transport compagny) prediction for a typical week in 2035 (VOLT scenario)

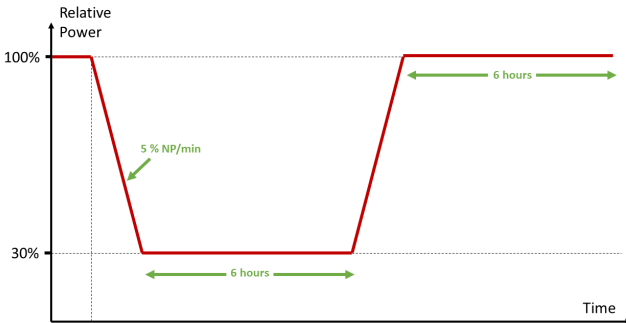


# Scenario of energetic transition in France



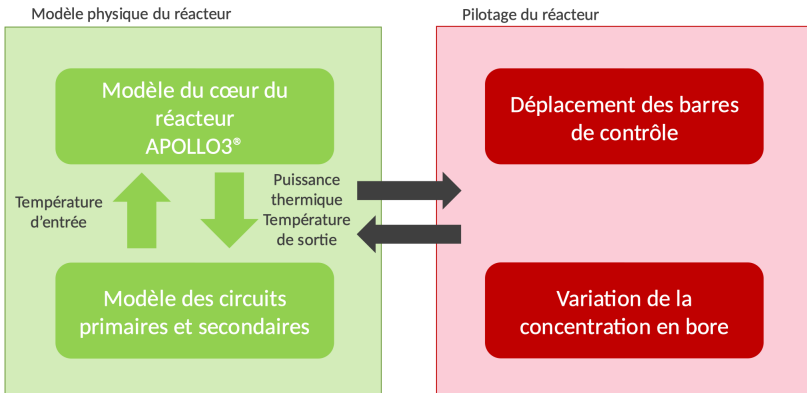
RTE (french electricity transport compagny) prediction for a typical week in 2035 (VOLT scenario)

# Target production transient



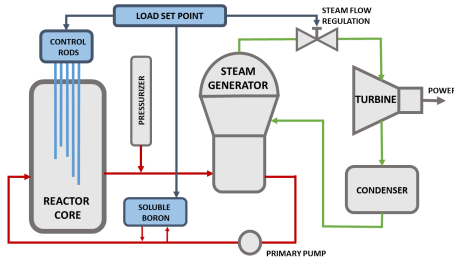
24h of production : most penalizing possible transient

# Multi-physic simulator



Around 10 minutes for the simulation of one transient  
(Now in 2022, 4 reactors, and potentially 40 min of simulation...)

# Optimization problem



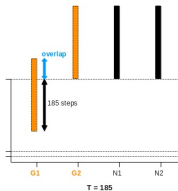
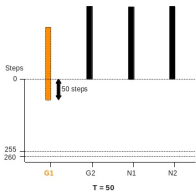
## Possible criteria

More than 7 criteria can be used :  
related to cost, safety, and stability

## Available control parameters

- Power Shimming Rods :  
Overlap (x3)  
Speed control (x4)
- Temperature Regulation Rods :  
maneuvering band (x1)

Search space size  $\approx 10^{12}$



# Fitness landscape analysis : offline model

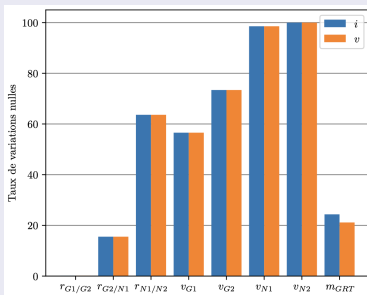
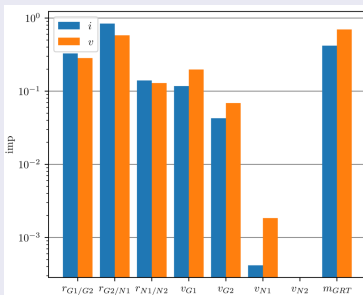
Using random walk sampling

## Objective correlation

	$v$	$N_R$	$C_{IPG}$	$\Delta T$	$f_T$	$i$	$F_v$
$v$	1.0	-0.75	-0.91	-0.14	0.12	-0.06	-0.01
$N_R$	-0.75	1.0	0.7	0.68	0.17	0.14	0.4
$C_{IPG}$	-0.91	0.7	1.0	0.12	-0.17	-0.02	-0.1
$\Delta T$	-0.14	0.68	0.12	1.0	0.7	0.51	0.79
$f_T$	0.12	0.17	-0.17	0.7	1.0	0.83	0.72
$i$	-0.06	0.14	-0.02	0.51	0.83	1.0	0.58
$F_v$	-0.01	0.4	-0.1	0.79	0.72	0.58	1.0

2 groups are highly correlated

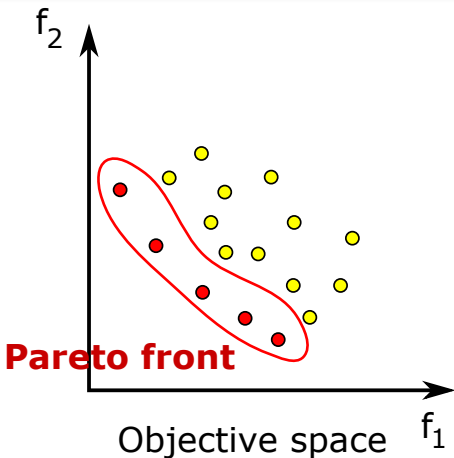
## Variable importance



Reduce the dimension to 6, and better understanding of the system

Allow to tune the mutation parameters

# Multiobjective optimization



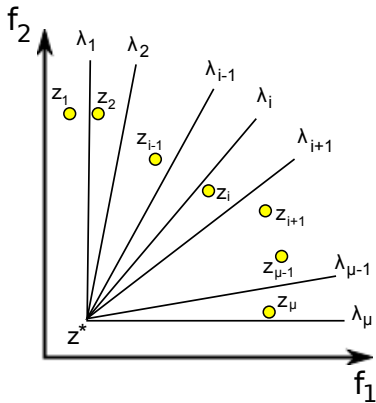
## Goal

No a priori on the order/importance of the objectives,  
Decision a posteriori based on the optimal Pareto solutions.

# MOEA/D : Multi-Obj. Evo. Algo. based on Decomposition

A lot of MO algo. :

Pareto based (NSGAII,...), indicator based (IBEA,...), and



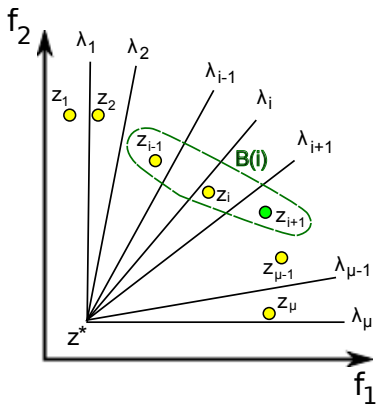
Population at iteration  $t$

- One solution  $x_i$  for each sub pb.  $i$  of direction  $\lambda_i$
- Scalar function  $g$  :  
Weighted Tchebycheff
- Representation of solutions in objective space :  $z_i = g(x_i | \lambda_i, z_i^*)$
- Same reference point for all sub-pb.  $z^* = z_1^* = \dots = z_\mu^*$
- Neighborhood size  $\#B(i) = T = 3$

# MOEA/D : Multi-Obj. Evo. Algo. based on Decomposition

A lot of MO algo. :

Pareto based (NSGAII,...), indicator based (IBEA,...), and



From the neigh.  $B(i)$  of sub-pb.  $i$ ,  
 $x_{i+1}$  is selected

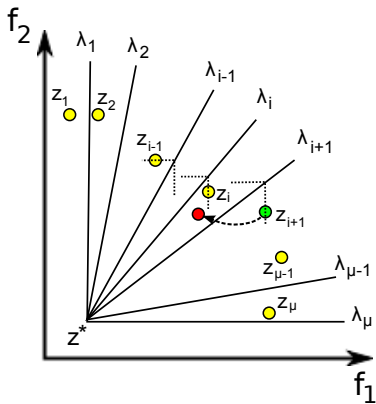
- One solution  $x_i$  for each sub pb.  $i$  of direction  $\lambda_i$
- Scalar function  $g$  :  
Weighted Tchebycheff
- Representation of solutions in objective space :  $z_i = g(x_i | \lambda_i, z_i^*)$
- Same reference point for all sub-pb.  $z^* = z_1^* = \dots = z_\mu^*$
- Neighborhood size  $\#B(i) = T = 3$



# MOEA/D : Multi-Obj. Evo. Algo. based on Decomposition

A lot of MO algo. :

Pareto based (NSGAII,...), indicator based (IBEA,...), and



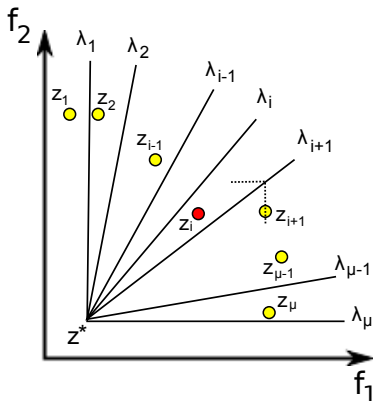
The mutated solution  $y$  is created

- One solution  $x_i$  for each sub pb.  $i$  of direction  $\lambda_i$
- Scalar function  $g$  :  
Weighted Tchebycheff
- Representation of solutions in objective space :  $z_i = g(x_i | \lambda_i, z_i^*)$
- Same reference point for all sub-pb.  $z^* = z_1^* = \dots = z_\mu^*$
- Neighborhood size  $\#B(i) = T = 3$

# MOEA/D : Multi-Obj. Evo. Algo. based on Decomposition

A lot of MO algo. :

Pareto based (NSGAII,...), indicator based (IBEA,...), and



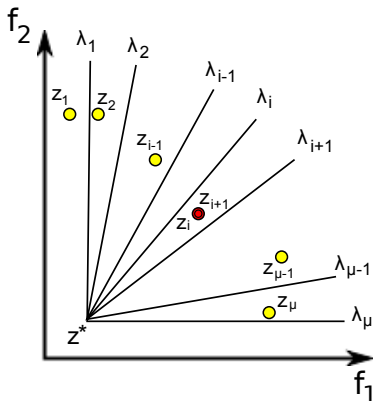
According to scalar function,  
 $y$  is worse than  $x_{i-1}$ ,  
 $y$  is better than  $x_i$  and replaces it.

- One solution  $x_i$  for each sub pb.  $i$  of direction  $\lambda_i$
- Scalar function  $g$  :  
Weighted Tchebycheff
- Representation of solutions in objective space :  $z_i = g(x_i | \lambda_i, z_i^*)$
- Same reference point for all sub-pb.  $z^* = z_1^* = \dots = z_\mu^*$
- Neighborhood size  $\#B(i) = T = 3$

# MOEA/D : Multi-Obj. Evo. Algo. based on Decomposition

A lot of MO algo. :

Pareto based (NSGAII,...), indicator based (IBEA,...), and



According to scalar function,  
 $y$  is also better than  $x_{i+1}$   
 and replaces it for the next iteration.

- One solution  $x_i$  for each sub pb.  $i$  of direction  $\lambda_i$
- Scalar function  $g$  :  
 Weighted Tchebycheff
- Representation of solutions in objective space :  $z_i = g(x_i | \lambda_i, z_i^*)$
- Same reference point for all sub-pb.  $z^* = z_1^* = \dots = z_\mu^*$
- Neighborhood size  $\#B(i) = T = 3$

# Massively parallel algorithm

## Optimization of problems based on expensive simulation

- Relevant tuning of parameters of the algorithm
- Surrogate model
- Parallel computing

Here,

Simulation for one burnup : 10 min

Simulation of 4 burnups (life cycle) : 40min

## Massive parallel system (HPC)

Algorithms for the TGCC (GENCI Projet)

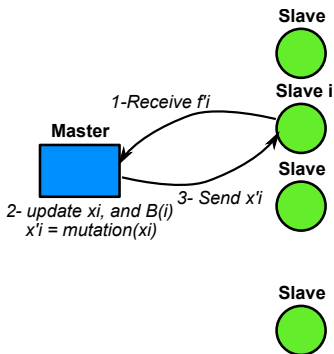
2 500 000 hours of available computation

1008 cores for 24h of computation.



# Asynchronous MOEA/D

## Master-slaves architecture



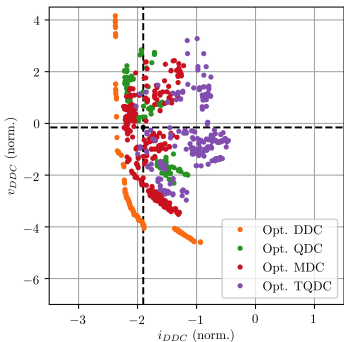
### Algorithm on Master

```
{ $x_1, \dots, x_\lambda$ }  $\leftarrow$  Initialization()  
for  $i = 1.. \lambda$  do  
    Send (Non-blocking)  $x_i$  to slave  $S_i$   
end for  
repeat  
    if there is a pending mess. from  $S_i$  then  
        Receive fitness  $f'_i$  of  $x'_i$  from  $S_i$   
        Update  $x_i$ , and  $x_j \in B(i)$  with  $(x'_i, f'_i)$   
         $x'_i \leftarrow \text{mutation}(x_i)$   
        Send (Non-blocking)  $x'_i$  to slave  $S_i$   
    end if  
until time limit
```

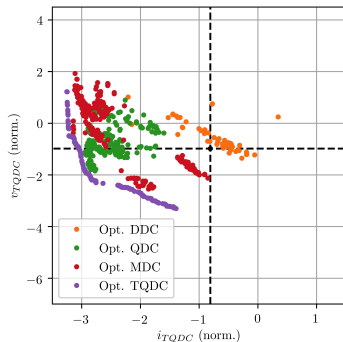
V. Drouet, S. Verel, and J-M. Do. "Surrogate-assisted asynchronous multiobjective algorithm for nuclear power plant operations.", Gecco 2020. [4]

# Results at different burnups

At the beginning of exploitation



At the end of exploitation



Optimization on the whole cycle is necessary

# Asynchronous MOEA/D with surrogate model

## Algorithm on Master

```
{ $x_1, \dots, x_\lambda$ } ← Initialization()
for  $i = 1.. \lambda$  do
  Send (Non-blocking)  $x_i$  to slave  $S_i$ 
end for
repeat
  if there is a pending mess. from  $S_i$  then
    Receive fitness  $f'_i$  of  $x'_i$  from  $S_i$ 
    Add ( $x'_i, f'_i$ ) to sample  $S$ 
    Update  $x_i$ , and  $x_j \in B(i)$  with ( $x'_i, f'_i$ )
    Train model  $M$  with Sample  $S$ 
    if  $|S| < N_{start}$  then
       $x'_i \leftarrow \text{mutation}(x_i)$ 
    else
      Select  $x'_i$  using model  $M$ 
    end if
    Send (Non-blocking)  $x'_i$  to slave  $S_i$ 
  end if
until time limit
```

## Surrogate model

Random forest

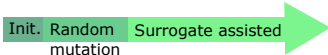
Offline tuning using data

Acceleration of  
convergence :

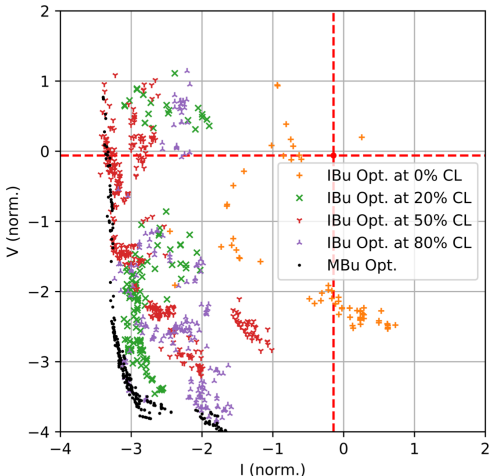
Double the prob. of  
improv.

Surrogate model can be  
misleading  
(poor accuracy at the  
begin.) :

Init. Random mutation      Surrogate assisted



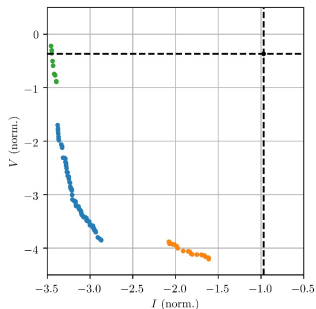
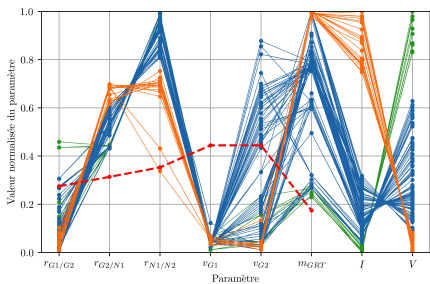
# Results on whole exploitation cycle



Reduce Volume of effluent, and Instability (axial offset)  
from current setting



# Analysis of decision variables



A postero interpretation of the candidate solutions on Pareto front

# Conclusion

## Optimization, machine learning techniques to solve design problems with digital twins

- Main tools :
  - Analysis fitness landscape to understand pb., and tune algo.
  - Use surrogate models, to accelerate the search, and bring an algebraic model
  - Parallel, and distributed computation
- A good algorithm is a tradeoff between :
  - Final decision making
  - Search space dimension, and its properties
  - Computation time, and power.
- Digital twins, and AI offers a lot of perspectives
  - How to combine different methods?
  - How to better understand systems? ...



Ricardo Baptista and Matthias Poloczek.

Bayesian optimization of combinatorial structures.

*In International Conference on Machine Learning*, pages 462–471. PMLR, 2018.



Thomas Bartz-Beielstein and Martin Zaefferer.

Model-based methods for continuous and discrete global optimization.

*Applied Soft Computing*, 55 :154–167, 2017.



Francisco Chicano, Darrell Whitley, Gabriela Ochoa, and Renato Tinós.

Optimizing one million variable nk landscapes by hybridizing deterministic recombination and local search.

*In Proceedings of the genetic and evolutionary computation conference*, pages 753–760, 2017.



V Drouet, Sébastien Verel, and J-M Do.

Surrogate-assisted asynchronous multiobjective algorithm for nuclear power plant operations.

In *Proceedings of the 2020 genetic and evolutionary computation conference*, pages 1073–1081, 2020.



V. Drouet, Sébastien Verel, and J.-M. Do.

Surrogate-assisted asynchronous multiobjective algorithm for nuclear power plant operations.

In *2020 Genetic and Evolutionary Computation Conference (GECCO '20)*, GECCO '20 : Proceedings of the 2020 Genetic and Evolutionary Computation Conference, pages 1073–1081, Cancún, Mexico, July 2020. ACM.



Valentin Drouet, Jean-Michel Do, and Sébastien Verel.

Optimization of load-follow operations of a 1300MW pressurized water reactor using evolutionary algorithms.

In M. Margulis and P. Blaise, editors, *International Conference on Physics of Reactors : Transition to a Scalable Nuclear Future (PHYSOR 2020)*, volume 247, page 11001, Cambridge, United Kingdom, March 2020.



Valentin Drouet, Jean-Michel Do, Sébastien Verel, and Jean-Charles Le Pallec.

Design of a simulator oriented PWR model and optimization of load-follow operations.

*In International Congress on Advances in nuclear Power Plants (ICAPP), Juan-les-pins, France, May 2019.*



Firas Hamze, Darryl C Jacob, Andrew J Ochoa, Dilina Perera, Wenlong Wang, and Helmut G Katzgraber.

From near to eternity : spin-glass planting, tiling puzzles, and constraint-satisfaction problems.

*Physical Review E*, 97(4) :043303, 2018.



Donald R Jones, Matthias Schonlau, and William J Welch.

Efficient global optimization of expensive black-box functions.

*Journal of Global optimization*, 13(4) :455–492, 1998.



Florian Leprêtre, Cyril Fonlupt, Sébastien Verel, and Virginie Marion.

SIALAC Benchmark : On the design of adaptive algorithms for traffic lights problems.

GECCO 2018, July 2018.

Poster.



Florian Leprêtre, Cyril Fonlupt, Sébastien Verel, and Virginie Marion.

Combinatorial Surrogate-Assisted Optimization for Bus Stops Spacing Problem.

In *Biennial International Conference on Artificial Evolution (EA 2019)*, Mulhouse, France, 2019.



Florian Leprêtre, Cyril Fonlupt, Sébastien Verel, Virginie Marion, Rolando Armas, Hernán Aguirre, and Kiyoshi Tanaka.

Fitness landscapes analysis and adaptive algorithms design for traffic lights optimization on sialac benchmark.

*Applied Soft Computing*, 85 :105869, 2019.



Florian Leprêtre, Sébastien Verel, Cyril Fonlupt, and Virginie Marion.

Walsh functions as surrogate model for pseudo-boolean optimization problems.

In *The Genetic and Evolutionary Computation Conference (GECCO 2019)*, Proceedings of the Genetic and Evolutionary Computation Conference, pages 303–311, Prague, Czech Republic, July 2019. ACM.



Mathieu Muniglia, Jean-Michel Do, Le Pallec Jean-Charles, Hubert Gard, Sébastien Verel, and S. David.

A Multi-Physics PWR Model for the Load Following.

In *International Congress on Advances in Nuclear Power Plants (ICAPP)*, San Francisco, United States, April 2016.



Mathieu Muniglia, Le Pallec Jean-Charles, Jean-Michel Do, and Sebastien Verel.

Design of a load following management for a PWR reactor using an optimization method.

In *MC 2017 - International Conference on Mathematics & Computational Methods Applied to Nuclear Science &*

*Engineering, International Conference on Mathematics & Computational Methods Applied to Nuclear Science & Engineering, Jeju, South Korea, April 2017.*



Mathieu Muniglia, Sébastien Verel, Le Pallec Jean-Charles, and Jean-Michel Do.

Massive asynchronous master-worker EA for nuclear reactor optimization : a fitness landscape perspective.

*In Genetic and Evolutionary Computation Conference (GECCO 2017), GECCO '17 Proceedings of the Genetic and Evolutionary Computation Conference Companion, Berlin, Germany, July 2017.*



Mathieu Muniglia, Sébastien Verel, Jean-Charles Le Pallec, and Jean-Michel Do.

A Fitness Landscape View on the Tuning of an Asynchronous Master-Worker EA for Nuclear Reactor Design.

*In International Conference on Artificial Evolution (Evolution Artificielle), pages 30–46, Paris, France, October 2017.*





Nestor V Queipo, Raphael T Haftka, Wei Shyy, Tushar Goel, Rajkumar Vaidyanathan, and P Kevin Tucker.

Surrogate-based analysis and optimization.

*Progress in aerospace sciences*, 41(1) :1–28, 2005.



Sébastien Verel, Bilel Derbel, Arnaud Liefoghe, Hernan Aguirre, and Kiyoshi Tanaka.

A surrogate model based on Walsh decomposition for pseudo-boolean functions.

In *International Conference on Parallel Problem Solving from Nature (PPSN 2018)*, volume 11102 of *Lecture Notes in Computer Science*, pages 181–193, Coimbra, Portugal, September 2018.



Martin Zaefferer, Jörg Stork, Martina Frieze, Andreas Fischbach, Boris Naujoks, and Thomas Bartz-Beielstein.

Efficient global optimization for combinatorial problems.

In *Proceedings of the 2014 annual conference on genetic and evolutionary computation*, pages 871–878, 2014.