On the Impact of Multiobjective Scalarizing Functions

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Multiobjective Optimization

Scenario: Multiobjective Optimization

Approximation of the Pareto front sought with good quality wrt indicator

Recent Interest in Decomposition Based Algorithms

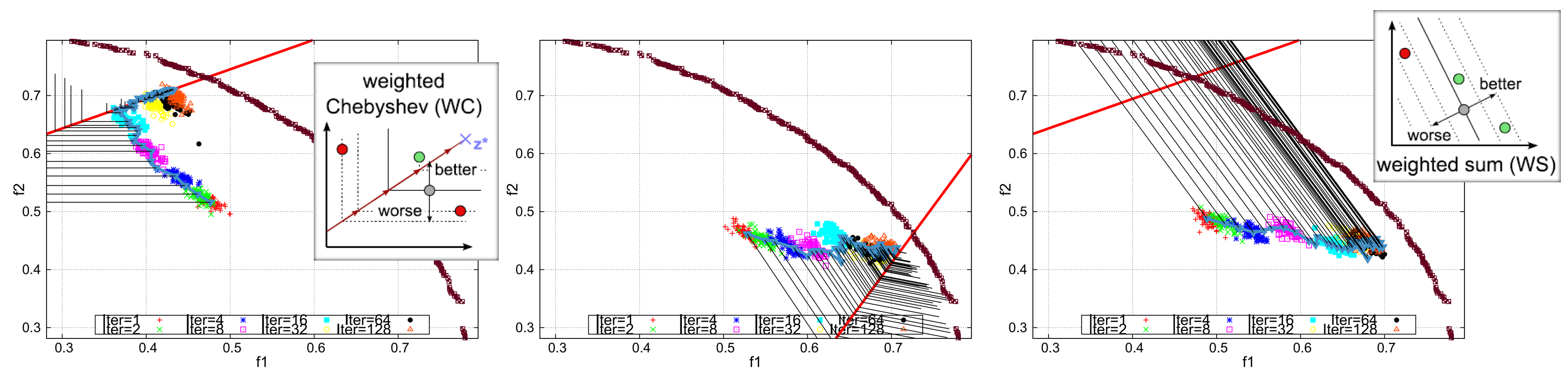
Transform multiobjective problem into several single-objective problems via scalarizing functions

examples are MOEA/D variants and MSOPS

Our Interest

Understand those algorithms in order to further improve them

Decompositon



$$\mathsf{S}_{\text{gen}}(z) = \alpha \cdot \max\{\lambda_1 \cdot |\bar{z}_1 - z_1|, \lambda_2 \cdot |\bar{z}_2 - z_2|\} + \varepsilon\,(w_1 \cdot |\bar{z}_1 - z_1| + w_2 \cdot |\bar{z}_2 - z_2|)$$

What Do We Do?

Empirical investigation

on the impact of the scalarizing function's parameters on the final distribution of points (direction and distance to Pareto front)

Abstraction from Complicated Algorithms

no crossover, no interactions, no archive

Simple Model

independent (1+lambda)-EAs on rhoMNK landscapes

Experimental Details

bi-objective case only (so far)

search: directions uniformly chosen in angle space

$$\mathsf{S}_{\text{norm}}(z) = (1-\varepsilon)\mathsf{T}(z) + \varepsilon \mathsf{WS}(z)$$

$$\lambda_i = 1/d_i \qquad w_1 = \cos(\delta) \quad w_2 = \sin(\delta)$$

$$\delta = \arctan(d_1/d_2)$$

$$\mathsf{S}_{\text{aug}}(z) = \mathsf{T}(z) + \varepsilon\,(|\bar{z}_1 - z_1| + |\bar{z}_2 - z_2|)$$

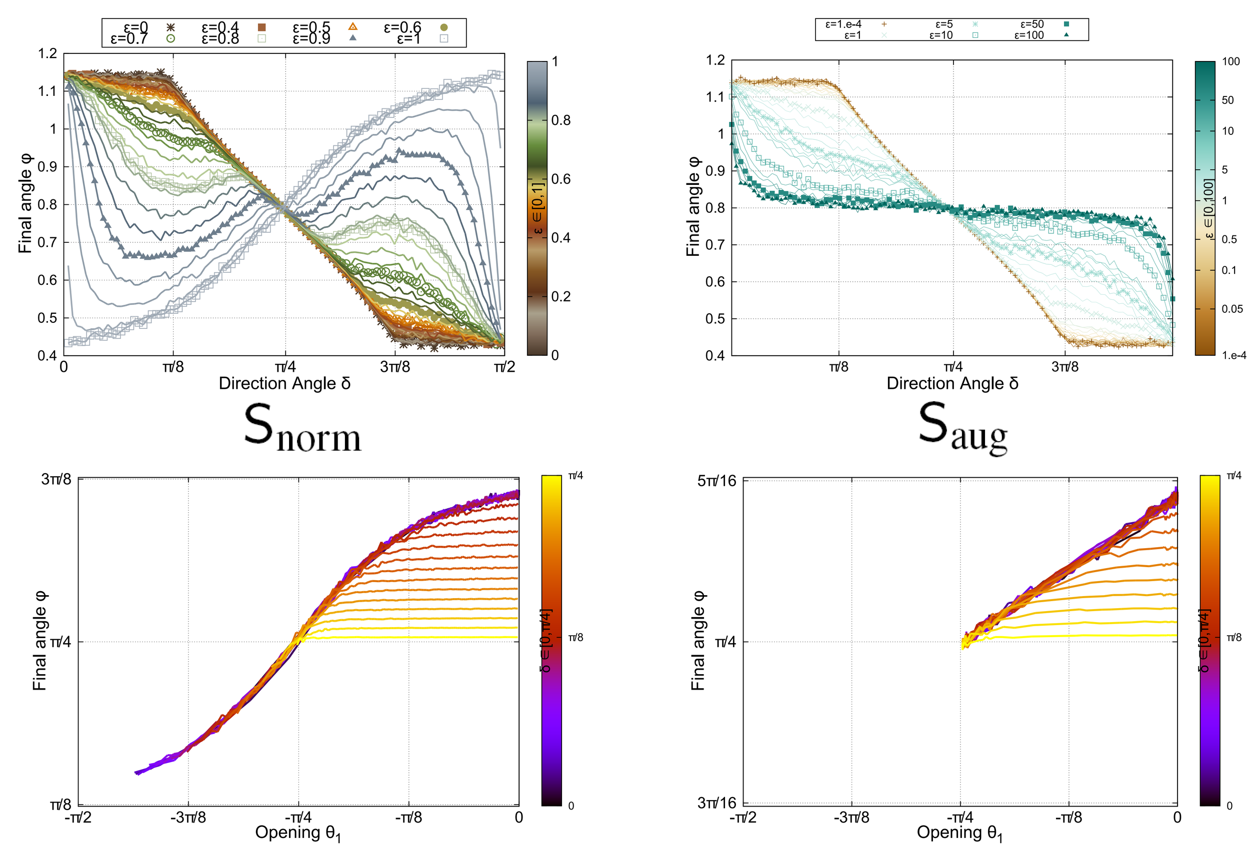
Table 2. Parameter setting.

scalarizing functions	ρMNK-landscapes	$(1+\lambda)$-EA
$\bar{z} = (1,1)$	$\rho \in \{-0.9, -0.8, \dots, 0.0, \dots, 0.9\}$	$\lambda = n$
$\delta = j \cdot 10^{-2} \cdot \frac{\pi}{2},\ j \in [\![1,99]\!]$	$m = 2$	bit-flip rate $= 1/n$
S_{norm}: $\varepsilon = \ell \cdot 10^{-2};\ \ell \in [\![0,100]\!]$	$n = 128$	stopped after
S_{aug}: $\varepsilon = \ell \cdot 10^{-k};\ \ell \in [\![0,10]\!]; k \in [\![-1,2]\!]$	$k = 4$	n iterations

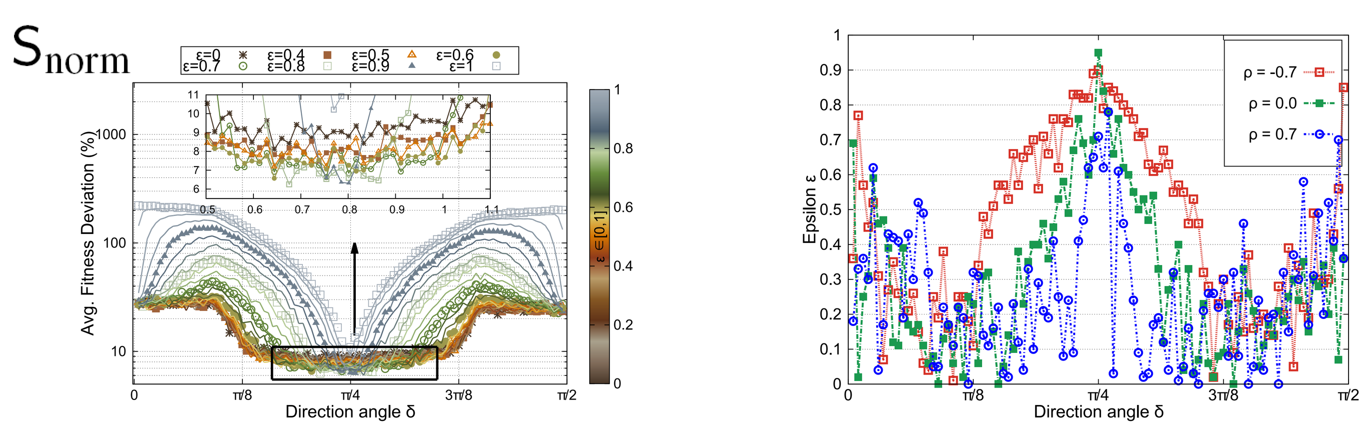
Single Search Behavior

Diversity: Final Angle

Proposition 1. *Let $\bar{z}$ be a utopian point, $\lambda_1, \lambda_2, w_1$, and $w_2 > 0$ scalar weighting coefficients, $\alpha \geq 0$ and $\varepsilon \geq 0$, where at least one of the latter two is positive. Then, the polar angles between the equi-utility lines of S_{gen} and the f_1-axis are $\theta_1 = \arctan(-\frac{\varepsilon w_1}{\alpha\lambda_2 + \varepsilon w_2})$ and $\theta_2 = \frac{\pi}{2} + \arctan(\frac{\varepsilon w_2}{\alpha\lambda_1 + \varepsilon w_1})$.*



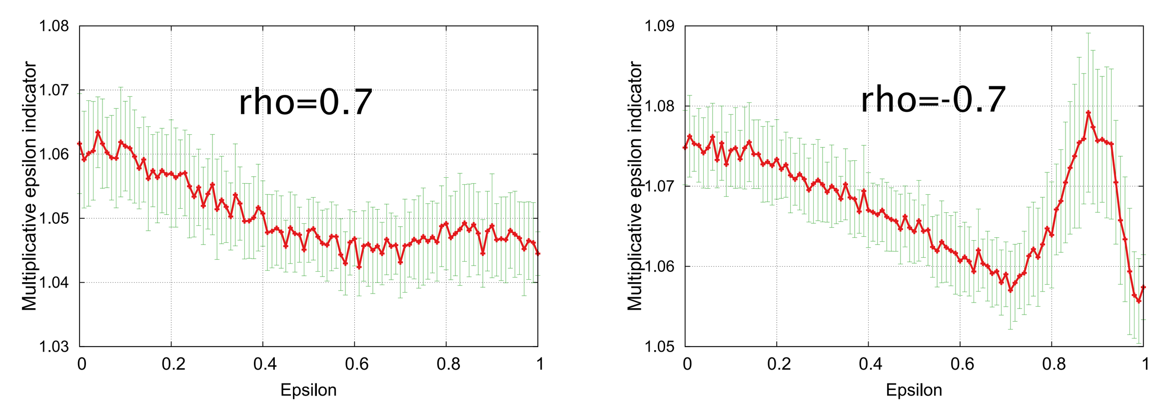
Convergence: Relative Deviation to Best



Global Search Behavior

Investigation of the impact of epsilon on the final quality indicator values when independent searches along 100 fixed directions are performed

Multiplicative Epsilon Indicator



Hypervolume Difference

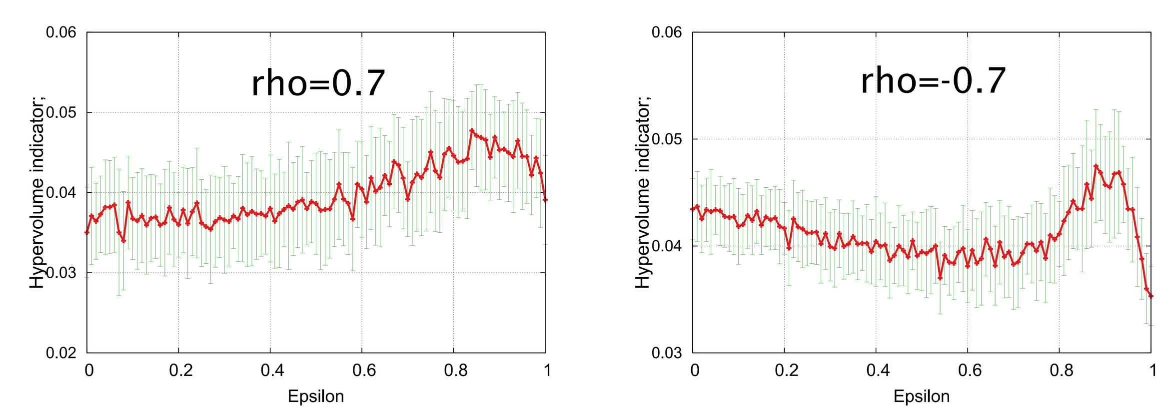
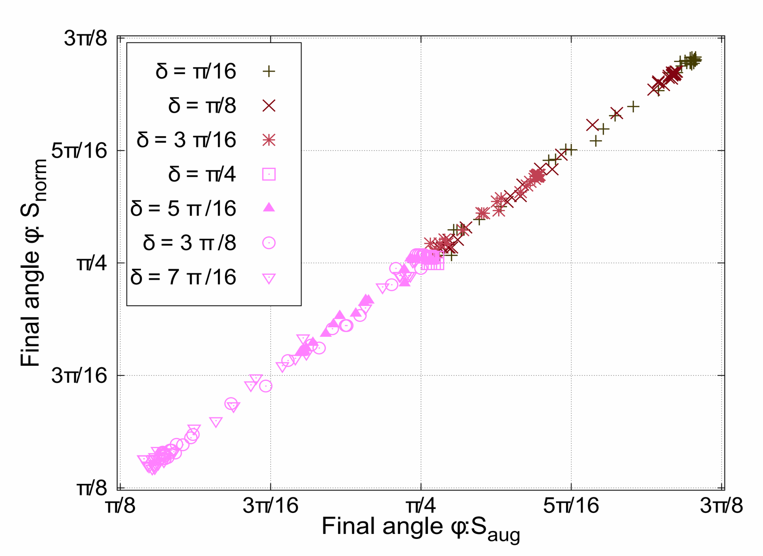


Table 3. Comparaison of WS, T, and non-uniform $\mathsf{S}^{\varepsilon^\star}_{\text{norm}}$ and $\mathsf{S}^{\varepsilon^\star}_{\text{aug}}$ configured with ε-values giving the best deviation w.r.t every direction. The number in braces shows the number of other algorithms that statistically outperform the algorithm under consideration w.r.t. a given indicator and a Mann-Whitney signed-rank statistical test with a p-value of 0.05 (the lower, the better).

	Avg. hypervolume difference ($\times 10^{-1}$)				Avg. multiplicative epsilon			
ρ	WS	T	$\mathsf{S}^{\varepsilon^\star}_{\text{norm}}$	$\mathsf{S}^{\varepsilon^\star}_{\text{aug}}$	WS	T	$\mathsf{S}^{\varepsilon^\star}_{\text{norm}}$	$\mathsf{S}^{\varepsilon^\star}_{\text{aug}}$
−0.7	0.353 (2)	0.434 (3)	0.324 (**0**)	0.307 (**0**)	1.057 (**0**)	1.075 (3)	1.059 (**0**)	1.057 (**0**)
0.0	0.418 (2)	0.458 (3)	0.357 (1)	0.322 (**0**)	1.056 (**0**)	1.084 (3)	1.062 (1)	1.064 (1)
0.7	0.391 (3)	0.350 (2)	0.303 (**0**)	0.292 (**0**)	1.044 (**0**)	1.062 (3)	1.047 (1)	1.047 (1)

Conclusions



An extensive empirical study shed more light on the impact of scalarizing functions within decomposition-based evolutionary multiobjective optimization.

It is fundamentally the opening of the lines of equal function values that explicitly guides the search towards a specific region of the objective space

Open(ing) (Re)search Lines

1. **Improving existing**
   e.g. non-uniform scalarizing functions
2. **Adapting the opening angles**
3. **Variation operators and problem-specific issues**
   do the observed findings also hold for other problems and operators?
4. **Theoretical Modeling**

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